

Linear forms in logarithms: the problem, the trivial bounds, and effectivity

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Two integers can be very large and still differ by one. To study a difference such as $2^x - 3^y$, divide by one of the powers. The question becomes how close $2^x 3^{-y}$ can be to 1. Taking logarithms changes multiplication into addition, but introduces a second question: how much cancellation can occur in $x \log 2 - y \log 3$?

This lesson develops the elementary estimates that make the question precise. It proves the passage between the multiplicative and additive forms, a height bound, a box-principle construction of small forms, and an explicit finiteness consequence of a polynomial lower bound. Its height prerequisite is the written lesson *Heights of algebraic numbers* in *Transcendental numbers*: Theorem 2.4 supplies the normalization, Proposition 2.5 the product and sum inequalities, and Theorem 2.8 the one-place Liouville inequality. The freely accessible treatments [Waldschmidt 1992], [Waldschmidt 2000] and [Evertse 2019] provide the comparisons discussed below. The logarithm methods of Baker and later authors are credited in the references.

1. Start with a Diophantine equation

If $2^x - 3^y = k$, then

$$2^x 3^{-y} - 1 = k 3^{-y}.$$

For fixed nonzero k , the right side is exponentially small as y grows. A lower bound that decreases only as a power of $\max(x, y)$ must eventually contradict this identity. This is the central mechanism of effective applications.

Let $\alpha_1, \dots, \alpha_n$ be nonzero algebraic numbers, let $b_1, \dots, b_n$ be integers, and choose logarithms ℓ_j with $e^{\ell_j} = \alpha_j$. Put

$$P = \prod_{j=1}^n \alpha_j^{b_j}, \quad \Delta = P - 1, \quad \Lambda = \sum_{j=1}^n b_j \ell_j, \quad B = \max\{3, |b_1|, \dots, |b_n|\}.$$

Thus $e^\Lambda = P$. In particular,

$$\Delta = 0 \iff \Lambda \in 2\pi i\mathbb{Z}.$$

If the α_j are positive real numbers and the ℓ_j are real logarithms, this reduces to $\Delta = 0 \iff \Lambda = 0$. A family is *multiplicatively independent* if $P = 1$ forces every $b_j = 0$. For positive real numbers this is exactly the rational linear independence of their real logarithms.

For 2 and 3, independence follows from unique factorization: $2^u 3^v = 1$ implies $u = v = 0$. More generally, independence of positive rationals can be checked by linear algebra on the vectors of their prime valuations. For example, 2, 3, 6 are dependent even though no two of them are powers of each other.

An *inhomogeneous* logarithmic form has the shape

$$\beta_0 + \beta_1 \ell_1 + \cdots + \beta_n \ell_n, \quad \beta_j \in \overline{\mathbb{Q}}.$$

It need not exponentiate to an algebraic number. Its exponential is $e^{\beta_0} \prod_j \alpha_j^{\beta_j}$, where each algebraic power means $e^{\beta_j \ell_j}$ for the chosen logarithm. For instance, $\pi + \log 2 = -i \log(-1) + \log 2$ is a homogeneous algebraic-coefficient logarithmic form if $\log(-1) = i\pi$.

2. Which logarithm is small?

Our principal logarithm has imaginary part in $(-\pi, \pi]$. We only use its holomorphic restriction near 1, so its convention on the negative real axis creates no ambiguity in the following estimates.

Lemma 1.1 (local comparison). If $|w| < 1/2$, then

$$|\operatorname{Log}(1+w)| \leq 2|w|.$$

If $|z| \leq 1/2$, then

$$|e^z - 1| \leq 2|z|.$$

Proof. The line segment from 1 to $1+w$ stays in the right half-plane. Hence

$$\operatorname{Log}(1+w) = \int_0^1 \frac{w}{1+tw} dt.$$

The integrand has absolute value at most $|w|/(1-|w|) \leq 2|w|$. Likewise

$$e^z - 1 = z \int_0^1 e^{tz} dt,$$

and $e^{|z|} \leq e^{1/2} < 2$. This proves both estimates. $\square$

Lemma 1.2 (branch correction). Take principal logarithms of the α_j . If $|\Delta| < 1/2$, there is an even integer b_0 such that

$$|b_0| \leq \sum_j |b_j| + 1 \leq nB + 1, \quad \left| b_0 \log(-1) + \sum_j b_j \log \alpha_j \right| \leq 2|\Delta|.$$

Proof. Set $u = \operatorname{Log}(1 + \Delta)$. Since $e^u = e^\Delta$, the elementary description of the kernel of the exponential gives $\Lambda - u = 2\pi i k$ for an integer k . Choose $b_0 = -2k$. Then the corrected form equals u , so Lemma 1.1 gives its bound. Moreover,

$$\pi|b_0| = |\operatorname{Im}(\Lambda - u)| \leq \pi \sum_j |b_j| + |\operatorname{Im} u|.$$

The number $1 + \Delta$ has positive real part, so $|\operatorname{Im} u| < \pi/2$. The claimed integer bound follows. $\square$

The correction cannot be omitted. If $\alpha_1 = \alpha_2 = -1$ and $b_1 = b_2 = 1$, then $P = 1$ but $\Lambda = 2\pi i$. There are examples with a small nonzero Δ too: take $\alpha_1 = -1$, $\alpha_2 = -(1 + 1/N)$, and both coefficients equal to 1. Then $\Delta = 1/N$, while $\Lambda = \log(1 + 1/N) + 2\pi i$.

For positive real bases, Λ is real and no correction is needed. For complex bases, smallness of $P - 1$ means closeness of Λ to the period lattice, not necessarily to its origin.

3. What arithmetic alone gives

We use the absolute logarithmic Weil height h . For a degree- d number γ , whose primitive minimal polynomial has positive leading coefficient a and roots $\gamma_1, \dots, \gamma_d$, our normalization is

$$h(\gamma) = \frac{1}{d} \left(\log a + \sum_{r=1}^d \log \max\{1, |\gamma_r|\} \right).$$

The prerequisite height facts are

$$h(uv) \leq h(u) + h(v), \quad h(u^{-1}) = h(u), \quad h(u + v) \leq h(u) + h(v) + \log 2,$$

and the one-place Liouville inequality: if $0 \neq u \in K \subset \mathbb{C}$ and $[K : \mathbb{Q}] = D$, then

$$\log |u| \geq -Dh(u).$$

The written internal proof provider is *Heights of algebraic numbers*, Theorem 2.8, together with Theorem 2.4 and Proposition 2.5. Its statement covers every number field and every nonzero algebraic element, with this absolute normalization. [Waldschmidt 1992, Chapter 3] gives a freely accessible complementary treatment with the same absolute-height convention.

Proposition 1.3 (elementary lower bound). If $\Delta \neq 0$ and $K = \mathbb{Q}(\alpha_1, \dots, \alpha_n)$ has degree D , then

$$\log |\Delta| \geq -D \left(\log 2 + \sum_j |b_j| h(\alpha_j) \right) \geq -D \left(\log 2 + B \sum_j h(\alpha_j) \right).$$

Proof. Repeated multiplication and inversion give $h(P) \leq \sum_j |b_j| h(\alpha_j)$. Since $h(1) = 0$, we have $h(P - 1) \leq h(P) + \log 2$. Apply Liouville's inequality to the nonzero element $P - 1$ of K . $\square$

There is a sharper elementary estimate for rationals. Write $\alpha_j = u_j/v_j$ in lowest terms, with nonzero integers u_j, v_j , and put $A_j = \max\{|u_j|, |v_j|\}$. Before any cancellation, write $P = U/V$ with integers $U, V \neq 0$. The denominator satisfies $|V| \leq \prod_j A_j^{|b_j|}$. If $P \neq 1$, the integer $U - V$ is nonzero, so

$$|\Delta| = \frac{|U - V|}{|V|} \geq \prod_j A_j^{-|b_j|}.$$

For $\Delta = 2^m 3^{-n} - 1$, with $m, n \geq 0$, one can use the actual denominator 3^n , obtaining $|\Delta| \geq 3^{-n}$. This improves the uniform denominator estimate $2^{-m} 3^{-n}$, but still falls exponentially with n .

For example, $3^5/2^8 - 1 = -13/256$. The actual denominator gives $13/256 \geq 1/256$; the uniform product estimate gives only $1/(3^5 2^8)$. The problem is not that the elementary bound is false. It allows far more cancellation than the exponents usually produce.

4. Why no fixed positive lower bound works

Lemma 1.4 (box principle). Let $n \geq 2$, let $a_1, \dots, a_n \geq 2$ be integers, and set $A = \max_j a_j$. For every integer $B > 2n \log A$, there are integers b_j such that

$$0 < \max_j |b_j| \leq B, \quad \left| \prod_j a_j^{b_j} - 1 \right| \leq \frac{2n \log A}{B^{n-1}}.$$

Proof. Consider the $(B+1)^n$ labelled points

$$s(c) = \sum_j c_j \log a_j, \quad c \in \{0, \dots, B\}^n,$$

in an interval of length at most $nB \log A$. If two labels give the same point, their difference supplies a nonzero vector with product exactly 1, which proves the assertion. Otherwise, sort the points. One of the $(B+1)^n - 1$ successive gaps is at most

$$\frac{nB \log A}{(B+1)^n - 1} \leq \frac{n \log A}{B^{n-1}}.$$

Subtract the corresponding labels. The resulting nonzero vector has coefficients of absolute value at most B , and its logarithmic form has absolute value at most the displayed quantity. Since $n \geq 2$ and $B > 2n \log A$, this is less than $1/2$. Lemma 1.1 finishes the proof. $\square$

For multiplicatively independent bases, the products constructed here are different from 1. Their distances from 1 tend to zero as $B \rightarrow \infty$. The same argument works for positive rationals: the points lie in an interval of length at most $B \sum_j |\log \alpha_j|$. Thus there cannot be a positive lower bound independent of the coefficients.

Combining the elementary lower bound and the box principle leaves a wide range:

$$e^{-cB} \quad \text{versus} \quad B^{-(n-1)}.$$

The first holds for every nonzero form; the second describes forms that exist for arbitrarily large coefficient bounds. A theorem of the shape $|\Delta| \geq B^{-C}$, with a fixed C , closes the most important part of that range. It does not assert that every coefficient vector nearly attains the bound.

5. From a polynomial bound to a finite search

Proposition 1.5. Suppose an explicitly known $C > 0$ satisfies

$$|2^x 3^{-y} - 1| \geq \max\{3, x, y\}^{-C}$$

whenever $x, y \geq 0$ and $2^x 3^{-y} \neq 1$. For a nonzero integer k , every solution of $2^x - 3^y = k$ satisfies

$$\max\{x, y\} \leq \max\{3, R(C, k)\},$$

$$R(C, k) = \left(\frac{C + \sqrt{C^2 + 4 \log 2 \log(2|k|)}}{2 \log 2} \right)^2.$$

Proof. Put $B = \max\{3, x, y\}$. The equation and the assumed bound give

$$3^y \leq |k| B^C, \quad 2^x \leq 3^y + |k| \leq 2|k| B^C.$$

If $\max(x, y) \geq 3$, then $B = \max(x, y)$ and $2^B \leq \max(2^x, 3^y) \leq 2|k|B^C$. Taking logarithms yields

$$B \log 2 \leq \log(2|k|) + C \log B.$$

For $t \geq 1$, $\log t \leq \sqrt{t}$: differentiation shows the maximum of $\log t/\sqrt{t}$ is $2/e < 1$. With $s = \sqrt{B}$, the preceding inequality implies

$$(\log 2)s^2 - Cs - \log(2|k|) \leq 0.$$

The positive root bounds s , giving $B \leq R(C, k)$. The remaining solutions have both exponents below 3. $\square$

This deliberately generous bound is enough to make a finite search possible. Later reductions replace large initial bounds by much smaller ones. If C is merely known to exist, Proposition 1.5 proves finiteness but does not specify the search range.

General differences of powers

Normalizing by the larger power gives a symmetric estimate. This is the argument of [Evertse 2019], Corollary 5.5; here we retain a prefactor, include zero exponents, and supply the resulting search bound.

Corollary (general differences of powers). Fix integers $a, b \geq 2$. Suppose known constants $0 < c \leq 1$ and $C > 0$ satisfy

$$|a^u b^v - 1| \geq c \max\{3, |u|, |v|\}^{-C}$$

for every integer pair (u, v) with $a^u b^v \neq 1$. For $m, n \geq 0$ with $a^m \neq b^n$, put $B = \max\{3, m, n\}$. Then

$$|a^m - b^n| \geq c \max\{a^m, b^n\} B^{-C}.$$

In particular, if $a^m - b^n = k \neq 0$, put

$$T = \log(|k|/c),$$

$$R_* = \left(\frac{C + \sqrt{C^2 + 4T \log 2}}{2 \log 2} \right)^2.$$

Then

$$\max(m, n) \leq \max\{2, R_*\}.$$

Proof. If $a^m > b^n$, apply the assumed estimate to $a^{-m} b^n$ and multiply by a^m . In the other case, apply it to $a^m b^{-n}$ and multiply by b^n . The coefficient bound is B in either orientation. This proves the first inequality.

When $\max(m, n) \geq 3$, at least one exponent equals B , so $\max(a^m, b^n) \geq 2^B$. Consequently

$$B \log 2 \leq \log(|k|/c) + C \log B.$$

Use $\log B \leq \sqrt{B}$ as in Proposition 1.5 and solve the quadratic inequality in $\sqrt{B}$. Its constant term is nonpositive, since $|k| \geq 1$ and $c \leq 1$. If both exponents are below three, their maximum is at most two. These cases give the stated bound. $\square$

The fixed-base estimate (5.47), proved later in *Effective lower bounds II: proof of Baker's theorem*, supplies an effective C with $c = 1$. Its proof follows the complete effective theorem in that lesson. Neither that estimate nor the corollary requires multiplicative independence. They apply, for example, to $4^m - 8^n = k$, provided the difference is nonzero. An elementary valuation argument solves one such example in Exercise 6.

6. Effectivity and the shape of the later theorems

A constant is *effectively computable from given data* if the proof supplies a terminating procedure producing a valid value from those data. A formula is one way to do this. A finite algorithm is another. An ineffective existence proof supplies neither, even if its conclusion is true.

Here is a concrete contrast. For $n \geq 2$ multiplicatively independent positive integers $a_1, \dots, a_n$, Gelfond's many-logarithm result says that, for every $\delta > 0$, there exists $c_\delta > 0$ such that

$$|a_1^{b_1} \cdots a_n^{b_n} - 1| \geq c_\delta e^{-\delta B}$$

for nonzero coefficient vectors, where $B = \max\{3, |b_j|\}$. The quoted result is ineffective in c_δ . Gelfond's earlier two-logarithm estimate gives, for each $\varepsilon > 0$, an effective $c_\varepsilon > 0$ in a lower bound

$$|a_1^{b_1} a_2^{b_2} - 1| \geq c_\varepsilon \exp(-(\log B)^{5+\varepsilon}).$$

These historical attributions and the distinction between the original effective and ineffective methods are discussed in [Waldschmidt 2000], Section 1.2, especially Theorem 1.9, and [Waldschmidt 1992], Introduction. Both free treatments state this historical comparison for integer bases. The stronger fixed-base polynomial estimate in this course applies to positive rationals as well. The following calculation proves both weaker numerical conclusions in that full setting with effective constants.

Lemma 1.6 (weaker bounds from a polynomial estimate). Suppose $|\Delta| \geq aB^{-C}$, with known $a, C > 0$ and $B \geq 3$. For every $\delta > 0$ and every $q > 1$,

$$|\Delta| \geq a \exp(C - C \log(C/\delta)) e^{-\delta B},$$

and

$$|\Delta| \geq a \exp\left(-(q-1)(C/q)^{q/(q-1)}\right) \exp(-(\log B)^q).$$

Proof. On $x > 0$, the function $\delta x - C \log x$ has its minimum at $x = C/\delta$, where its value is $C - C \log(C/\delta)$. Thus $B^{-C} e^{\delta B}$ is at least the first exponential constant. On $t \geq 0$, the minimum of $t^q - Ct$ occurs at $t = (C/q)^{1/(q-1)}$ and equals $-(q-1)(C/q)^{q/(q-1)}$. Use $t = \log B$ for the second bound. $\square$

With $q = 5 + \varepsilon$, the second formula has the two-logarithm shape above. For fixed positive rational bases, the polynomial estimate is proved in the effective-bound lessons, specifically (5.47), from the full effective Baker theorem and the local comparison of Section 2. Consequently both weaker numerical bounds admit effective constants whenever the product is not one; the historical many-logarithm proof did not supply them. Lemma 1.6 states exactly where that later input is used. The elementary arguments in Sections 2–4 do not depend on it.

The following table describes shapes, not interchangeable theorem statements. The heights, branches and nonvanishing conditions must be specified afresh whenever a theorem is applied. Write $\Omega = \prod_j \log A_j$, where the admissible A_j depend on the theorem.

Result	Characteristic conclusion	Treatment in this course
Baker's qualitative theorem	Rationally independent logarithms remain independent over the algebraic numbers, even after adjoining 1	Proved by an auxiliary function and extrapolation

Result	Characteristic conclusion	Treatment in this course
Baker's algebraic-coefficient measure	A nonzero form has size $> B^{-C}$, with effectively computable C and bounded algebraic degrees	Proved in <i>Effective lower bounds II: proof of Baker's theorem</i> ; B bounds coefficient heights
Feldman's polynomial refinement	Integer-coefficient multiplicative forms satisfy $ \Delta \geq B^{-C}$	Fixed-base consequence proved in <i>Effective lower bounds II: proof of Baker's theorem</i> , Section 10; individual-height estimates treated in the lesson on many logarithms
Baker's product-height refinement	An exponent involving $\Omega \log \Omega$	Treated in <i>Linear forms in many logarithms: the modern estimates and how to use them</i> , alongside the subsequent refinements
Philippon–Waldschmidt, Wüstholz, and Baker–Wüstholz	A lower bound whose exponent involves $\Omega \log B$	Statements, conditions and deductions given in that same lesson on many logarithms
Laurent's determinant method	Two-logarithm estimates involving $(\log B')^2$	General parameter theorem proved in <i>Two logarithms I: an interpolation determinant</i> ; sharper explicit families in <i>Two logarithms II: explicit lower bounds</i>
Matveev's estimate	Explicit product-height bounds with improved dependence on the number of logarithms	Theorem 8.6 and its applications in the lesson on many logarithms
Yu and Bugeaud–Laurent	Upper bounds for a normalized p -adic valuation	Proofs planned in <i>Two p-adic logarithms and Yu's estimates</i>

The algebraic-coefficient and integer-coefficient versions use different size parameters. Keeping this distinction prevents a transcendence measure from being mistaken for an integer-exponent estimate.

One further distinction matters. For $\alpha = 1 + 1/N$,

$$\frac{1}{N+1} \leq \log \left(1 + \frac{1}{N} \right) \leq \frac{1}{N}, \quad h(\alpha) = \log(N+1).$$

The inequalities follow by integrating $1/(1+t)$ from 0 to $1/N$. The logarithm is small even though the height is large. Refined estimates with a parameter E exploit this discrepancy. A height parameter cannot be replaced by the size of the logarithm alone.

7. Exercises

Exercise 1 (easy). Determine exactly when $\Delta = 0$ in terms of Λ , and explain what changes for positive real bases. Give an example showing why principal logarithms of negative bases can require a period correction.

Exercise 2 (easy). For each $1 \leq n \leq 10$, choose the integer $m \geq 0$ minimizing $|2^m - 3^n|$. Compare $|2^m 3^{-n} - 1|$ with both elementary rational lower bounds from Section 3.

Exercise 3 (medium). Repeat the box-principle proof for multiplicatively independent positive rationals $\alpha_1, \dots, \alpha_n$, with $n \geq 2$. Give a sufficient lower threshold for B and prove that the nonzero distances from 1 accumulate at zero.

Exercise 4 (medium). In Lemma 1.4, explain why the count is $(B+1)^n$, why a repeated point is harmless there, and why multiplicative independence is needed when one wants the constructed distance to be nonzero. Derive the exact successive-gap bound before estimating it by $n \log A/B^{n-1}$.

Exercise 5 (hard). Assume the lower bound in Proposition 1.5. Prove its explicit search bound, including solutions where one exponent is zero. Then derive an explicit bound when the hypothesis is weakened to $|2^x 3^{-y} - 1| \geq cB^{-C}$, with known $0 < c \leq 1$.

Exercise 6 (medium). The bases 4 and 8 are multiplicatively dependent. Solve $4^m - 8^n = 56$ for integers $m, n \geq 0$, using the exact power of 2 dividing each side. Explain why the general corollary also applies to this equation.

8. Solutions

Solution 1. Since $e^\Lambda = 1 + \Delta$, the kernel $2\pi i\mathbb{Z}$ of the complex exponential gives the exact criterion. Real logarithms give a real Λ ; its intersection with that kernel is $\{0\}$. With two bases equal to -1 and both coefficients equal to 1, the principal logarithmic form is $2\pi i$, although $\Delta = 0$.

Solution 2. For a fixed 3^n , the closest power of 2 is one of the two powers immediately bracketing it. All others are farther away by monotonicity. Checking those two integers gives:

n	nearest exponent m	$ 2^m - 3^n $	exact normalized distance
1	1 or 2	1	1/3
2	3	1	1/9
3	5	5	5/27
4	6	17	17/81
5	8	13	13/243
6	9	217	217/729
7	11	139	139/2187
8	13	1631	1631/6561
9	14	3299	3299/19683
10	16	6487	6487/59049

Each normalized distance is at least 3^{-n} , because its numerator is a positive integer. It is consequently also at least the weaker bound $2^{-m}3^{-n}$. The first two rows attain the actual-denominator bound. Small coefficient values need not show a smooth trend: the polynomial lower bounds are uniform restrictions on cancellation, not monotonicity claims.

Solution 3. Put $S = \sum_j |\log \alpha_j| > 0$. The sums indexed by $\{0, \dots, B\}^n$ lie between $B \sum_{\log \alpha_j < 0} \log \alpha_j$ and $B \sum_{\log \alpha_j > 0} \log \alpha_j$, an interval of length BS . Independence makes the sums distinct. The smallest successive gap is at most $BS/((B+1)^n - 1) \leq S/B^{n-1}$. For an integer $B > 2S$, this is below $1/2$. Subtract labels and use Lemma 1.1 to obtain a nonzero multiplicative distance at most $2S/B^{n-1}$. These bounds tend to zero. If the corresponding coefficient vectors belonged to a fixed finite set, their positive distances would have a positive minimum, a contradiction. Thus the construction supplies arbitrarily small nonzero distances.

Solution 4. Each coordinate has $B+1$ choices, including both endpoints. There are therefore $(B+1)^n$ labelled sums. A repeated sum yields a nonzero difference of labels whose product is exactly 1, meeting the weak inequality in the lemma. Under independence, repetition is impossible. For

distinct sums $s_1 < \dots < s_N$, the $N - 1$ successive gaps add to $s_N - s_1 \leq nB \log A$; at least one gap is at most $nB \log A / ((B + 1)^n - 1)$. Finally $(B + 1)^n - 1 \geq B^n$, giving the stated simpler estimate.

Solution 5. For the original hypothesis, Section 5 gives all steps. They use only $x, y \geq 0$, so zero exponents are included. With the weakened hypothesis, the equation gives $3^y \leq (|k|/c)B^C$ and $2^x \leq (|k|/c)B^C + |k| \leq (2|k|/c)B^C$. Replace $\log(2|k|)$ throughout the quadratic argument by $\log(2|k|/c)$. The explicit bound is

$$\max(x, y) \leq \max \left\{ 3, \left(\frac{C + \sqrt{C^2 + 4 \log 2 \log(2|k|/c)}}{2 \log 2} \right)^2 \right\}.$$

Solution 6. The difference is positive, so $2m > 3n$, and

$$56 = 2^{3n}(2^{2m-3n} - 1).$$

The factor in parentheses is odd. Since $56 = 2^3 \cdot 7$, its exact power of two forces $3n = 3$, hence $n = 1$. The odd factor then gives $2^{2m-3} = 8$, so $m = 3$. Conversely, $4^3 - 8 = 56$. This includes the possible zero-exponent cases: the valuation already forces $n = 1$, and positivity forces $m > 0$. The corollary requires only a nonzero difference and a fixed-base lower bound; a multiplicative relation among the bases does not violate either condition.

Prerequisites and continuation

The sole arithmetic prerequisite beyond elementary rational arithmetic is the absolute-height theory specified in Section 3, supplied by the written *Heights of algebraic numbers* lesson in *Transcendental numbers*. The local comparison, branch correction, rational and height lower bounds, box-principle construction, finite-search calculations and Lemma 1.6 are proved here. The general difference-of-powers corollary is proved here under its stated lower-bound hypothesis; the later estimate (5.47) supplies that hypothesis. The table in Section 6 locates the different theorem shapes and their conditions. The next two lessons prove the qualitative Baker theorem before the course turns to effective constants.

References

- [Waldschmidt 1992] Michel Waldschmidt, *Linear Independence of Logarithms of Algebraic Numbers*, IMSc Report 116, 1992, revised text dated 2016. Introduction and Chapter 3. Author’s text.
- [Waldschmidt 2000] Michel Waldschmidt, *Diophantine Approximation on Linear Algebraic Groups*, Springer, 2000. Author’s full text. Section 1.2 explains the elementary denominator, box-principle and historical effectivity comparisons; Chapter 3 supplies the height framework.
- [Evertse 2019] Jan-Hendrik Evertse, *Diophantine Equations*, Chapter 5, “Linear forms in logarithms,” 2019, author’s chapter. Corollaries 5.3 and 5.5 explain the branch correction and normalized difference-of-powers argument. The effective logarithm theorem itself is stated there; this course supplies its proof in its effective-bound lessons.

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